Bayesian Account of Diachronic Coherence and Dogmatic Attitudes

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Abstract

According to Bayesian account of diachronic coherence, one should respond to new evidence only by conditionalizing on the evidence. Here I argue that this view is indefensible because probabilities equal to zero or one cannot change through Bayesian conditionalization. Accordingly, an agent won't change her extreme subjective probabilities — those equal to zero or one — no matter what defeating evidence she might later receive. That is, extreme subjective probabilities become dogmatic attitudes if one is to respond to new evidence only by Bayesian conditionalization. One may suggest that contingent propositions should never be assigned extreme probabilities, therefore, Bayesian account of diachronic coherence does not imply that dogmatic attitudes toward contingent propositions are occasionally admissible or required by rationality. In the paper, I challenge this suggestion and conclude that holding extreme subjective probabilities for contingent propositions is not problematic on its own but becomes so when combined with the assumption that Bayesian conditionalization is the only rational way of updating beliefs. I argue that this is not a reason for objecting to assigning extreme probabilities to contingent propositions, instead, it is a reason for objecting to Bayesian account of diachronic coherence.

Introduction

How should one respond to new evidence? Bayesian answer is by conditionalizing. Many would agree that learning calls for updating prior degrees of belief in one way or another. Yet, conditionalization is the only *rational* way of updating degrees of belief according to Bayesian account of diachronic coherence. Here I argue that this view is indefensible as it implies no evidence can alter extreme subjective probabilities of an agent — which are equal to zero or one.

Classical Bayesian conditionalization involves use of Bayes' theorem combined with Bayes' rule. It is applicable only when the evidence raises the subjective probability of a proposition up to one, as using Bayes' rule requires assigning probability one to some evidential proposition. Jeffrey's conditionalization replaces Bayes' rule with Jeffrey's rule. It thus provides a way of updating beliefs on an evidential proposition in which one invests only partial belief. Neither conditionalization, though, can accommodate evidential propositions with zero prior probability. Consequently — as I

demonstrate in the paper— subjective probabilities equal to zero or one cannot change through either way of conditionalization. Thus, Bayesian account of diachronic coherence implies that extreme subjective probabilities won't change no matter what defeating evidence one may later receive. That is to say, extreme subjective probabilities are rendered dogmatic attitudes, if one is to respond evidence only by Bayesian conditionalization. Bayesian account of diachronic coherence not only depicts such dogmatic attitudes as *rational* but implies that, once a proposition is assigned an extreme probability, rationality requires clinging to that attitude toward the relevant proposition at any rate. I will argue that this is an unacceptable implication for extreme subjective probabilities pertaining to contingent propositions — as they are always fallible— if not for any extreme subjective probability.

Supposedly, one should never assign probability one or zero to a contingent proposition anyway. In that case, Bayesian account of diachronic coherence would not imply that dogmatic attitudes toward contingent propositions are occasionally admissible or required by rationality. That is, its implications for extreme subjective probabilities would not necessarily trouble Bayesian account of diachronic coherence. I will consider some possible objections to assigning probability one or zero to contingent propositions and try to answer them. I will argue that these objections do not suffice to vindicate that contingent propositions should never be assigned extreme probabilities. Thus, Bayesian account of diachronic coherence cannot be defended by reserving extreme probabilities for noncontingent propositions. I will conclude that holding extreme subjective probabilities for contingent propositions is not a problematic attitude on its own but becomes so when combined with the assumption that Bayesian conditionalization is the only rational way of updating beliefs. I hold that this is not a reason to object to assigning extreme probabilities to contingent propositions, but rather, a reason to object to Bayesian account of diachronic coherence.

Bayesian Account of Diachronic Coherence and Conditionalization

According to Bayesian epistemology, beliefs come with degrees and degrees of belief can be represented by real numbers between 0 and 1— by probabilities. Accordingly, an agent's overall doxastic state at a particular time can be captured by a probability function, which takes propositions and gives real numbers in the unit interval as values (Ebert and Martin 2012a, p. 305). These values correspond to degrees of belief of the agent, namely, subjective probabilities. According to Bayesian approach one's subjective probabilities should conform to the probability calculus in order to be synchronically coherent (Hájek 2012b, p. 411). This is required by Bayesian account of rationality.

Bayesian account of rationality also requires diachronic coherence. Probability functions are specific to a particular time, so are subjective probabilities (Howson and Urbach 1996, p. 99). For diachronic coherence, subjective probabilities should change upon receiving new evidence — they should be updated. According to Bayesians, *the* coherent way of updating subjective probabilities is conditionalizing on the evidence. This process involves a shift of probability function. Conditionalizing on some evidence means giving up the prior probability function P and adopting a new one, P′. In this case, P(.) expresses a subjective probability before receiving the evidence and P′(.) expresses a subjective probability just after receiving the evidence.

Classical Bayesian conditionalization is regulated by Bayes' rule of conditionalization and Bayes' theorem. Bayes' rule relates P(.) and P′(.) (Strevens 2017). Assume that content of some evidence can be captured by an evidential proposition E. In that case, acquiring the evidence amounts to learning that E is true. For this case, Bayes' rule can be written as P′(.) = P(.|E). It dictates that posterior probability of a proposition just after learning E must be set equal to prior probability of that proposition conditional on E. In other words, one's subjective probability for a proposition, let it be H, conditional on some proposition E must become her unconditional subjective probability for H if and when she learns that E is true (Strevens 2017, p. 23). Bayes' theorem yields the probability of H conditional on E. The theorem is as follows:

$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$ (Howson and Urbach 1996, p. 99).

It can be derived from the ratio formula of conditional probability.

As Bayes' rule sets P′(.) equal to P(.|E), P′(E) equals one. Accordingly, classical Bayesian conditionalization calls for adopting a new probability function relative to which the evidence acquired has a probability of one. This is applicable only when one learns that some evidential proposition is true, that is, when evidence sends the subjective probability for some proposition to one. Yet, as Jeffrey points out, in some cases observation requires a new distribution of subjective probabilities but there is no evidential proposition which can be said to be learned (Jeffrey 1965, p. 165). Observation might raise the subjective probability for some proposition to some degree but not necessarily all the way to one (Strevens 2017, p. 24). Jeffrey's conditionalization lifts the requirement of assigning probability one to an evidential proposition, thereby providing a way to incorporate probabilistic evidence into conditionalization. Jeffrey's rule together with Bayes' theorem regulates Jeffrey's conditionalization. Jeffrey's rule is a generalization of Bayes' rule and can be written as:

P′(H) = P(H|E) x P′(E) + P(H|~E) x P′(~E) (Howson and Urbach 1996, p. 106).

Jeffrey's conditionalization thus involves giving up the prior probability function and adopting a new probability function P′ relative to which an evidential proposition E has a positive probability. It can be seen Jeffrey's rule reduces to Bayes' rule when probability one is assigned to proposition E — when P′(E) = 1 (Norton 2011, p. 432).

It should be noted that if some evidence is to be incorporated into doxastic system of an agent through conditionalization, the evidence must raise the probability of some proposition whose prior probability is other than zero. Neither classical Bayesian conditionalization nor Jeffrey's conditionalization allows conditionalizing on an evidential proposition with prior probability zero. As the value of posterior probability —P(H|E)— is given by Bayes' theorem, prior probability of E — P(E) — must be positive, otherwise P(H|E) goes undefined.

Bayesian account of diachronic coherence stipulates that one should respond to new evidence by conditionalizing only. But this amounts to not responding to evidence when it raises the probability of a proposition with a prior probability of zero — as requisite posterior probability goes undefined in these cases. This implies that no evidence can be incorporated into conditionalization if it suggests that a proposition which is assigned probability one is false or that a proposition which is assigned zero probability is true. Such evidence would have a prior subjective probability equal to zero. Besides, no evidence with a positive prior probability can alter the value of an extreme subjective probability even though it can be incorporated into conditionalization: No evidence can raise a subjective probability zero through conditionalization, just as no evidence can lower a subjective probability equal to one (Lewis 1986, p. 268). The former is obvious from Bayes' theorem. If prior probability of H is zero, its posterior probability will be zero, whenever it is defined, independent of the prior probability of evidence or its likelihood on H. As to subjective probabilities equal to one, it is overt that they won't change through conditionalization from another formulation of Bayes' theorem:[1]

$$P(H|E) = \frac{P(H) \times P(E|H)}{P(H) \times P(E|H) + P(\sim H) \times P(E|\sim H)}$$

When P(H) is one, P(~H) is zero. Thus, equation reduces to $\frac{P(H) \times P(E|H)}{P(H) \times P(E|H)}$ which is equal to one whenever defined. Plus, use of Jeffrey's rule instead of Bayes' rule would make no difference as when P(H|E) is one, P(H|E) x P′(E) + P(H|~E) x P′(~E) is also one and when P(H|E) is zero, P(H|E) x P′(E) + P(H|~E) x P′(~E) is zero as well. Thus, extreme

[1] To be sure, either formulation yields the same results whenever defined and goes undefined when prior probability of E — P(E)— is zero. Here I use a different formulation only because this formulation makes it easier to see that if P(H) is one, P(H|E) will be one as well, whatever the positive value P(E) might have.

subjective probabilities retain their values —zero or one— through Bayesian conditionalization no matter what evidence one may later receive. Accordingly, Bayesian account of diachronic coherence not only implies that clinging to extreme subjective probabilities is *rational* but that one should cling to those attitudes if she is to remain rational — even if further evidence suggests that she is or might be wrong. I hold that this implication is unacceptable, and thus, calls for rejecting the assumption that one should respond to new evidence only by conditionalization.

Possible Responses and Objections

It can be argued that one should stick to her beliefs in some propositions.[2] If X is a tautology, it seems acceptable to invest full belief in X regardless what evidence may suggest. Besides, what evidence could cast doubt on a tautology anyway? One may even find some virtue in sticking to logical necessities: If some evidence conflicts with such propositions, one might think that it is better to discard the evidence rather than questioning logical truths. Even if this view is accepted, sticking to subjective probabilities for contingent propositions is definitely no virtue. It is a dogmatic attitude — a rational one according to Bayesianism— and unacceptable given that any belief in a contingent proposition is fallible.

Precisely because extreme subjective probabilities won't change through conditionalization, David Lewis submits that initial *credence* functions should be *regular* (Lewis 1986, p. 267). That is, one should not hold any extreme subjective probability for a contingent proposition before receiving any evidence (Lewis 1986, p. 268). I won't argue here whether this constraint on initial subjective probabilities should be admitted. Even if one has only non-extreme subjective probabilities for contingent propositions initially, she might invest full belief in contingent propositions as she receives evidence: Content of the evidence can be captured by an evidential proposition in some cases and this evidential proposition can be assigned probability one. In other cases, evidence can raise the probability of some proposition(s) up to one, even if its content cannot be fully captured by a particular proposition. To be sure, as mentioned, assigning probability one to some evidential proposition is not required by Jeffrey's conditionalization, but this is not to say that it is precluded. Thus, even if Lewis' constraint on initial subjective probability function is admitted, one may invest full belief in some propositions by learning. That is, one may come to hold extreme subjective probabilities for contingent propositions by assigning probability one to evidential propositions. And these subjective probabilities are rendered dogmatic attitudes by

[2] No need to say, rejecting this view would imply no dogmatic attitude is acceptable. In that case, dogmatic attitudes toward contingent propositions are *a fortiori* unacceptable.

Bayesian account of diachronic coherence — they won't change no matter how strong defeating evidence one may later acquire. No need to say, this is also unacceptable since evidence is fallible and further evidence may defeat or, at least, cast doubt on previous evidence both in daily life and in scientific practice (Norton 2011, p. 432).

It can be argued that one should not assign probability one to any evidential proposition anyway, so there is no harm in rendering extreme subjective probabilities dogmatic attitudes. If this claim is accepted —together with Lewis's constraint on initial probability functions— extreme probabilities are reserved for noncontingent propositions. In that case, Bayesian account of diachronic coherence would not imply that clinging to fallible beliefs is occasionally admitted and required by rationality. In what follows, I consider some possible objections to assigning probability one to evidential propositions. I try to answer them in order to establish that holding extreme subjective probabilities for evidential propositions — or rather, contingent propositions in general— is not problematic on its own but when it is coupled with the idea that conditionalization is the only rational way of updating beliefs and with the commitment to Bayes' formula which goes undefined when an evidential proposition has a prior probability of zero. If this is the case, and I will argue that it is; Bayesian account of diachronic coherence cannot be defended by reserving extreme subjective probabilities for noncontingent propositions.

One may argue that contingent propositions should never be assigned probability one, because this would amount to assigning probability zero to propositions incompatible with it. Those propositions are possible and assigning probability zero to them would be treating them as impossible. This objection can be formulated in a stronger way by emphasising that a rational agent is aware that the evidential propositions are contingent. So, she is also aware that propositions incompatible with them are possible. Therefore, it would be incoherent to assign probability one to an evidential proposition, which is equivalent to assigning probability zero to some possible propositions.

For one thing, this objection assumes that assigning probability zero to a proposition amounts to treating it as impossible. I contend that this view is dubious. There are counter-examples against this assumption, showing that an event with zero probability might happen, that is, a zero-probability event is possible (Hájek 2012b, 416). For instance, any point on an idealized dartboard has zero probability of being hit (by the idealized dart) as there are uncountably many points. But we know that dart hits one of them. Another counterexample is the case of a fair coin landing heads every time when it is tossed infinitely many times. The probability of this event is zero and yet it is possible (Williamson 2007, pp. 173-180). Hence assigning probability zero to a

proposition need not imply treating it as impossible and, therefore, assigning probability one to a contingent proposition does not mean ignoring its contingency. Those examples show that probability zero does not necessarily imply impossibility, and thus, holding a subjective probability of zero for a proposition does not necessarily involve an attitude committed to impossibility of this proposition.

Supposedly, the events mentioned above do not have a zero probability but their probability is infinitesimal (Hájek 2012b, p. 416). That is to say, their probability is infinitely close to zero but not zero (Lewis 1986, p. 268). In that case, these counterexamples would not show that a proposition with zero probability is possible. Yet, even if these counterexamples are set aside and it is granted that probability zero means impossibility, neither formulation of the objection at hand would suffice to show that no contingent proposition should be assigned probability zero or one. They would, then, suffice to show that propositions about chancy events should not be assigned extreme probabilities. But this conclusion is not relevant to evidential propositions as evidential propositions don't concern chancy events. At any rate, evidential propositions are not modal claims; they capture the content of some experiential evidence. Assigning probability one to an evidential proposition does not imply that one holds that it is necessarily true, but just that she believes that it is the case. Thus, there is nothing in conflict with contingency in assigning probability one to an evidential proposition. Consequently, there is no incoherence in assigning probability one to a proposition which is known to be contingent.

One may go on to argue that contingent propositions should never be assigned probability one since they might be false. Even evidential propositions which one holds can be false given that evidence is fallible. Arguments against reliability of experience abound. In view of the fact that earlier evidence is later found to be dubious even in scientific practice, it is plausible to argue that no evidence is ever certain (Norton 2011, p. 432). Yet, I contend that fallibility is not a reason to refrain from assigning probability one to a proposition.

For one thing, so long as subjective probabilities can change through updating on evidence, there is nothing problematic about holding an inaccurate subjective probability. Openness to updating beliefs presumes that beliefs might be false and, by the same token, that subjective probabilities might be inaccurate. If defeating evidence can later change the subjective probability for an evidential proposition, fallibility of evidence is no reason to avoid assigning probability one to an evidential proposition. Surely, according to Bayesian account of diachronic coherence, once an extreme probability is assigned to a proposition, this subjective probability becomes final and won't change no matter what defeating evidence might be later acquired. Yet, this is a reason

for giving up Bayesian account of diachronic coherence — or at least for revising it— rather than a reason for refraining from assigning probability one to evidential propositions (Hájek 2012b, p. 421).

Hence, fallibility is rather a reason to abstain from absolute certainty as to contingent propositions. But absolute certainty and holding a subjective probability equal to one are different attitudes. A subjective probability equal to one does not necessarily imply an attitude of absolute certainty. To be sure subjective probabilities equal to one indeed imply absolute certainty if Bayesian conditionalization is regarded as the only way of changing beliefs (Williamson 2000, p. 214). This is because extreme subjective probabilities won't change through Bayesian conditionalization regardless of the evidence that might be acquired later. Yet again, this is a reason for objecting to Bayesian account of diachronic coherence, not for reserving extreme subjective probabilities for noncontingent propositions.

Conclusion

If one is to respond evidence only by Bayesian conditionalization, no evidence will alter extreme subjective probabilities. Accordingly, Bayesian account of diachronic coherence implies that rationality requires clinging to extreme subjective probabilities regardless what further evidence might suggest. I have argued that this implication is unacceptable for contingent propositions, even if it is accepted for noncontingent ones. I have also argued that Bayesian account of diachronic coherence cannot be defended by reserving extreme probabilities for noncontingent propositions as there is no reason to refrain from assigning probability one to evidential propositions. Holding extreme subjective probabilities for contingent propositions rendered problematic by the assumption that Bayesian conditionalization is the only rational way of updating beliefs. I contend that this is not a reason to avoid assigning extreme subjective probabilities to contingent propositions but a reason to object Bayesian account of diachronic coherence.

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